Examples for difference equations

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1. Solve the difference equation $y_{t+1} - 1.1y_t = 0$ by iteration for year 2, 3, 4 and 5, given the income in year 1 is 20,000 Bahts..

Solution. We have $y_{t+1} = 1.1y_t$, therefore

$$\begin{aligned} y_1 &= 20000 \\ y_2 &= 1.1(20000) = 22000 \\ y_3 &= 1.1(22000) = 24200 \\ y_4 &= 1.1(24200) = 26620 \\ y_5 &= 1.1(26620) = 29282 \end{aligned}$$

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2. Write out the solution of the difference equation $y_{t+1} - 1.1y_t = 0$ for $t = 1$, 2, 3, 4 and 5 in terms of y_1. Deduce the general expression for y_t in terms of y_1. Evaluate y_{40} given $y_t = 20000$.

Solution.

$$\begin{aligned} y_2 &= 1.1y_1 \\ y_3 &= 1.1y_2 = 1.1\,(1.1y_1) = (1.1)^2y_1 \\ y_4 &= 1.1y_3 = 1.1(1.1)^2y_1 = (1.1)^3y_1 \\ y_5 &= 1.1y_4 = 1.1(1.1)^3y_1 = (1.1)^4y_1 \end{aligned}$$

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In general,

$$y_t = (1.1)^{t-1}y_2$$

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$t = 40$, $y_1 = 20000$;

$$y_{40} = (1.1)^{39}20000 = 822895.56$$

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3. Find a general solution of the difference equation $y_{t+1} - 0.9y_t = 0$. If $y_2 = 100$, find the particular solution. Evaluate y_1, y_2, y_3, y_{20} and y_{50}.

Solution. The general form of the solution is $y_t = Aa^t$. Therefore $y_{t+1} = Aa^{t+1}$. Then the difference equation becomes

$$\begin{aligned} y_{t+1} - 0.9y_t &= 0 \\ Aa^{t+1} - 0.9a^t &= 0 \\ Aa^t(a - 0.9) &= 0 \end{aligned}$$

Since $A \neq 0$ and $a^t \neq 0$, therefore $a - 0.9 = 0$, that is $a = 0.9$. Thus the general solution is

$$y_t = A(0.9)^t$$

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From $y_2 = 100$;

$$\begin{aligned} y_2 &= A(0.9)^2 \\ 100 &= A(0.9)^2 \\ A &= 123.46 \end{aligned}$$

The particular solution is then

$$y_p = 123.46(0.9)^t$$

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Then we tabulate the required calculation,

t	y_t
1	$123.46(0.9) = 111.11$
2	$123.46(0.9)^2 = 100.00$
3	$123.46(0.9)^3 = 90.00$
20	$123.46(0.9)^{20} = 15.01$
50	$123.46(0.9)^{50} = 0.64$

Bibliography

Teresa Bradley and Paul Patton. *Essential mathematics for economics and business.* 2nd ed. 2002(1998)